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A NEW RANDOM NETWORK GENERATOR FOR ACTIVITY-ON-THE-NODE NETWORKS

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ABSTRACT

In this paper we describe a new random network generator *RanGen* for generating activity-on-the-node (AoN) networks and accompanying data for different classes of project scheduling problems. The objective is to construct random networks which satisfy preset values of the parameters used to control the hardness of a problem instance. Both parameters which are related to the network topology and resource-related parameters are implemented. The new network generator meets the shortcomings of former network generators since it employs a wide range of different parameters which have been shown to serve as possible predictors of the hardness of different project scheduling problems. Some of them have been implemented in former network generators while others have not. Copies of the computer program can be obtained from the authors.

Keywords: Networks/graphs; Problem generator; Complexity index; Resource constraints;

1 Introduction

Since the beginning of the research in project scheduling, activity networks (*AN*) have become an important tool to visualize different kinds of projects. Rapid progress regarding exact and suboptimal procedures has created the need to differentiate between easy and hard project scheduling instances. Therefore, one must possess a set of measures that discriminates between easy and hard instances and that acts as a predictor of the computational effort of the procedures.

Quite a number of measures have been proposed in the literature (Davis, 1975; Patterson, 1976). Recently, Herroelen and De Reyck (1999) have shown the occurrence of phase transitions in project scheduling problems and call the attention to the importance of measures with sufficient discriminatory power to allow for the observation of these dramatic changes in problem difficulty. We can distinguish between measures capturing information about the size and the topological structure of the network and measures which are related to the different resources allocated to the project. Patterson (1984) was the first to satisfy the need for a standard set of problems with varying degrees of difficulty. His testset contains problems taken from different sources in the literature. Unfortunately, the testset lacks a controlled design of several parameters and has been recently labelled as rather easy. Motivated by this fact, the recognition arose for the need for networks with controllable measures of complexity which has led to the construction of a few network generators and the generation of new standard datasets.

To the best of our knowledge, papers dealing with network generators for project scheduling problems are rather sparse. Demeulemeester et al. (1993) have developed a random generator for activity-on-the-arc (AoA) networks. These networks are so-called *strongly random* since they can be generated at random from the space of all feasible networks with a specified number of nodes and arcs. Besides the number of nodes and the number of arcs, no other characteristics can be specified for describing the network topology. The number of renewable resource types as well as the resource availabilities and requirements generation are constant or drawn from precoded distributions. Kolisch et al. (1995) describe *ProGen*, a network generator for activity-on-the-node (AoN) networks which takes into account network topology as well as resource-related characteristics. Schwindt (1995) extended *ProGen* to *ProGen/Max* which can handle three different types of resource-constrained project scheduling problems with minimal and maximal time lags. Agrawal et al. (1996) recognize the importance of the complexity index *CI* as a measure of network complexity and have developed an activity-on-the-arc network generator *DAGEN* for which this complexity measure can be set in advance. Neither of the networks generated by the last three generators can be called *strongly random* because they do not guarantee that the topology is a random selection from the space of all possible networks which satisfy the specified input parameters.

The aim of this paper is to present an AoN network generator which generates problem instances which span the full range of problem complexity (Elmaghraby and Herroelen, 1980). The generator uses a reliable set of complexity measures which have been shown in former studies to stand in clear and strong relation to the hardness of different project scheduling problems (Elmaghraby and Herroelen, 1980; De Reyck,

1995; De Reyck and Herroelen, 1996; Herroelen and De Reyck, 1999). The network generator also guarantees networks with a prespecified order strength *OS*. Moreover, we seem to satisfy the need for a random AoN network generator with prespecified values of the network complexity index *CI*, as mentioned by De Reyck (1995), De Reyck and Herroelen (1996), Demeulemeester et al. (1998) and Herroelen and De Reyck (1999).

The organisation of the paper is as follows. In section 2 we briefly review the most important conclusions from the literature on the use of network-based measures of complexity. Section 3 introduces a procedure for the generation and unique representation of all networks which satisfy a preset topological structure. Section 4 explains the basics of the network generator *RanGen*. In section 5 we try to gain additional insight in the relation between two network topology measures: the complexity index and the order strength. Section 6 reviews a number of existing resource measures and their implementation in our new generator. Section 7 provides a summary and overall conclusions.

2 Network topology measures

Contributions to measures of network complexity have received attention from researchers since the mid-sixties and emanate from studies in the area of activity networks, assembly line balancing and machine scheduling problems. Ideally, such measures should serve as predictors of the hardness of a problem as measured by the CPU-time. For a complete evaluation of contributions regarding these measures, we refer to Elmaghraby and Herroelen (1980).

Probably the best known measure for the topological structure of activity-on-the-arc networks is the *coefficient of network complexity* (*CNC*), defined by Pascoe (1966) as the number of arcs over the number of nodes, and redefined by Davies (1974) and Kaimann (1974, 1975). The measure has been adapted for activity-on-the-node problems by Davis (1975) as the number of direct arcs over the number of activities (nodes) and has been used in the network generator *ProGen* (Kolisch et al., 1995). Since the measure relies totally on the count of the activities and the direct arcs of the network and as it is easy to construct networks with an equal *CNC*-value but a different degree of difficulty, Elmaghraby and Herroelen (1980) questioned the usefulness of the suggested measure. De Reyck and Herroelen (1996) and Herroelen and De Reyck (1999) conclude that the correlation of the *CNC* with the so-called complexity index *CI* is responsible for a number of misinterpretations with respect to the explanatory power of the *CNC*. Indeed, Kolisch et al. (1995) and Alvarez-Valdes and Tamarit (1989) had revealed that resource-constrained project scheduling instances become easier with increasing values of the *CNC*, without considering the underlying effect of the complexity index. In conclusion, the *CNC*, by itself, fails to discriminate between easy and hard instances and can therefore not serve as a good measure for describing the impact of the network topology on the hardness of a project scheduling problem.

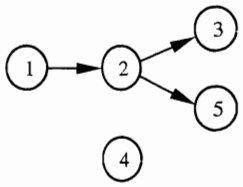
Another well-known measure of the topological structure of an AoN network is the *order strength*, *OS* (Mastor, 1970), defined as the number of precedence relations (including the transitive ones but not including the arcs connecting the dummy start or

end activity) divided by the theoretical maximum number of precedence relations ($n(n-1)/2$, where n denotes the number of nondummy activities in the network). It is sometimes referred to as the density (Kao and Queranne, 1982) or the restrictiveness *RT* (Thesen, 1977) and equals 1 minus the flexibility ratio (Dar-El, 1973). Herroelen and De Reyck (1999) conclude that the order strength *OS*, the density, the restrictiveness and the flexibility ratio constitute one and the same complexity measure. Schwindt (1995) uses the restrictiveness *RT* in the problem generator *ProGen/Max* and argues that this measure plays an important role in predicting the difficulty of different resource-constrained project scheduling problems. De Reyck (1995) verified and confirmed the conjecture that the *OS* outperforms the complexity index *CI* as a measure of network complexity for the resource-constrained project scheduling problem.

The complexity index *CI* was originally defined by Bein et al. (1992) for two-terminal acyclic activity-on-the-arc networks as the *reduction complexity*, i.e. the minimum number of node reductions which – along with series and parallel reductions – allow to reduce a two-terminal acyclic network to a single edge. As a consequence, the *CI* measures the closeness of a network to a series-parallel directed graph. Their approach for computing the reduction complexity consists of two steps. First, they construct the so-called complexity graph by means of a dominator and a reverse-dominator tree. Secondly, they determine the minimal node cover through the use of the maximum flow procedure by Ford and Fulkerson (1962). De Reyck and Herroelen (1996) adopted the reduction complexity as the definition of the complexity index *CI* of an activity network and have proven the complexity index to outperform other popular measures of performance, such as the *CNC*. On the other hand, they conclude that the *OS* outperforms the *CI*. These studies motivated the construction of an AoN problem generator for networks where both the order strength *OS* and the complexity index *CI* can be specified in advance. To the best of our knowledge, *DAGEN* (Agrawal et al., 1996) is the only generator which generates networks with prespecified complexity index. They construct problems in AoA format and do not take the order strength as a measure of network topology into account. Unfortunately, the generated networks are not strongly random.

In the next section we introduce an algorithm to represent networks in a unique fashion by means of a so-called standardized upper triangular matrix. In the succeeding section this enumerative procedure will be used in our construction method for generating networks with a specified order strength and complexity index.

3 A unique representation of networks

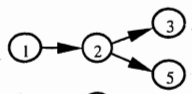


	2	3	4	5
1	1	1	-	1
2		1	-	1
3			-	-
4				-

Fig. 1. An example network with its Precedence Matrix representation *PM*

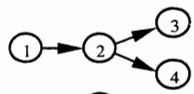
A project network in AoN format $G=(N,A)$ where the set of nodes, N, represents activities and the set of arcs, A, represents precedence constraints can be represented by an *upper triangular precedence relations matrix* without the diagonal as given in Fig. 1. This binary precedence matrix *PM* denotes whether or not a precedence relation exists between two nodes. If node j is a successor (either direct or transitive) of node i then $PM_{ij}=1$, otherwise it equals zero. Notice that in Fig. 1 activity 1 has three successors: arc (1,2) represents a direct precedence relation, while (1,3) and (1,5) denote transitive precedence relations. Activity 2 has two immediate successors: activities 3 and 5. In line with the literature, we add a dummy start activity s and a dummy end activity t to visualize the network.

The representation of a network as an upper triangular matrix *PM* has serious advantages. First, it is never possible to have activities with a smaller number than one of its predecessors and secondly, it leads to a very easy and precise calculation of the order strength *OS*. The number of elements in *PM*, either zero or one, denotes the maximal number of arcs in the considered network, while the number of ones denotes the number of precedence relations. From the definition of the *OS*, we have to divide this number of ones by the number of precedence relations to obtain the value for the order strength. Remark that the order strength in our example equals 5/10=0.5.



	2	3	4	5
1	1	1	-	1
2		1	-	1
3			-	-
4				-

(a)



	2	3	4	5
1	1	1	1	-
2		1	1	-
3			-	-
4				-

(b)

Fig. 2. Two identical networks with a different Precedence Matrix representation *PM*

As shown in Fig. 2, it is easy to construct networks with the same topological structure but a different precedence matrix *PM*. Although the two networks differ in

the successors of node 2, the topology of both networks is the same. Since we are only interested in networks with a different topological structure, we have to detect such similarities one way or another. In the next section we discuss a generally fast recursive procedure which transforms different matrices *PM* of networks with the same topological structure into one and the same standardized precedence matrix *SPM*.

3.1 The recursive algorithm

The recursive algorithm implicitly enumerates all possible representations of a network which satisfy a given topological structure, the unique representation of which will be specified by its *standardized precedence matrix SPM*.

To that purpose we assign a weight w_i to each node i of the network *AN* with a precedence matrix *PM* based on its number of successors and predecessors (both direct and transitive) and create an array *rea* by a recursive enumeration method in which the nodes are ranked according to specific criteria. This array *rea* (*recursively enumerated array*) will then be used to create the standardized precedence matrix *SPM* of the network *AN*. During the recursive enumeration procedure we create each time an eligible set of nodes *EN* for which all predecessor nodes are already enumerated. Among the set of eligible nodes *EN*, we choose the node with the highest weight w_i to include in the array *rea*, we update *EN* and continue our recursive enumeration method until we have enumerated (and ranked) all nodes of the network. The recursive method is a complete enumeration in the sense that, when tie breaks occur (nodes with an equal weight to choose from), we split the recursive enumeration method in a number of cases equal to the number of nodes with equal weight and continue our recursive enumeration method for each case, assuring that each possible rank order will be found.

Since each rank order given by its array *rea* corresponds to a network *AN* with the same topological structure, we can compute a *representation value rv* for each array *rea* (and its corresponding network), using the idea of binary digits. Each arc in the network has a value which is twice the value of the preceding arc in the precedence matrix *PM*. In doing so, we assure that each network can be identified in a unique way, similar to the binary representation of any integer number by a power of two. Among the possible rank orders found, we choose the one with the smallest value *rv* which leads, by means of rearranging the nodes, to the standardized precedence matrix *SPM*. Notice that the recursive search algorithm finds the minimal representation value *rv* under the condition that nodes with maximal weight are selected at each level of the recursion. It does therefore not guarantee an optimal solution, i.e. an overall minimal representation value.

Let *UN* denote the set of unenumerated nodes, *EN* the set of eligible nodes, *PM* the precedence matrix of the activity network *AN* in the AoN format, p the level of the recursive enumeration algorithm and *rea* the array in which we will rank the different nodes. $\overline{S}_i$ and $\overline{P}_i$ denote, respectively, the sets of successors and predecessors (including the transitive ones) of node $i \in PM$, while S_i and P_i denote the sets of their immediate successors and predecessors. Remark that the variable E is a local auxiliary

variable of the recursive procedure while *SN* denotes a set which stores the set of eligible nodes *EN* at each level of the recursion. The procedure to represent each network in a unique fashion, including the recursive enumeration algorithm, can be written as follows:

Procedure Unique_representation(*AN*);
  Initialize $UN=N$, $EN=\{i|P_i=\varnothing\}$, $rv=\infty$ and $\forall i\in N$, $w_i=2^{24}|\overline{S}_i|+2^{16}|S_i|+2^8|\overline{P}_i|+|P_i|$;
  Do *RECURSIVE_ENUMERATION*$(1,EN) \rightarrow rea$;
  Rearrange the Precedence Matrix *PM* according to *rea*.

RECURSIVE_ENUMERATION(*p*, *EN*)
If $p=n$ **then**
  $rea_i=p|i\in EN$;
  If $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} 2^{\{n(rea_i-1)-\frac{rea_i(1+rea_i)}{2}+rea_j-1\}} PM_{ij} < rv$ **then**
    save $rea_i|i\in N$ in the array *opt_rea* and update *rv*;
else
  $E=\{i|w_i = \max(w_j|j\in EN)\}$;
  $\forall i\in E$
    $SN=EN$, $EN=EN\backslash\{i\}$, $UN=UN\backslash\{i\}$ and $rea_i=p$;
    $EN=EN\cup\{j|j\in S_i$ and $P_j\cap UN=\varnothing\}$;
    RECURSIVE_ENUMERATION$(p+1, EN)$;
    $UN=UN\cup\{i\}$ and $EN=SN$;
Return;

In order to clarify the procedure as described above, we apply the unique representation algorithm to the precedence matrix *PM* of Fig. 1.

Table 1. Calculations of the weights w_i for each node

i	$\lvert\overline{S}_i\rvert$	$\lvert S_i\rvert$	$\lvert\overline{P}_i\rvert$	$\lvert P_i\rvert$	w_i
1	3	1	0	0	50,397,184
2	2	2	1	1	33,685,761
3	0	0	2	1	513
4	0	0	0	0	0
5	0	0	2	1	513

Initialize $UN=\{1,2,3,4,5\}$, $EN=\{1,4\}$, $rv=\infty$ and $w_1=50{,}397{,}184$, $w_2=33{,}685{,}761$, $w_3=513$, $w_4=0$ and $w_5=513$. The calculations of the weights w_i are shown in Table 1.
RECURSIVE_ENUMERATION$(1, \{1,4\})$;
$E=\{\mathbf{1}\}$;
$EN=\{4\}$, $UN=\{2,3,4,5\}$ and $rea_1=1$;
$EN=\{2,4\}$;
  RECURSIVE_ENUMERATION$(2, \{2,4\})$;
  $E=\{\mathbf{2}\}$;
  $EN=\{4\}$, $UN=\{3,4,5\}$ and $rea_2=2$;
  $EN=\{3,4,5\}$;

RECURSIVE_ENUMERATION(3, {3,4,5});
E={**3**,5};
EN={4,5}, *UN*={4,5} and rea_3=3;
EN={4,5};
RECURSIVE_ENUMERATION(4, {4,5});
E={**5**};
EN={4}, *UN*={4} and rea_5=4;
EN={4};
RECURSIVE_ENUMERATION(5, {4});
rea_4=5;
Since p=5 and $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} 2^{\{n(rea_i-1)-\frac{rea_i(1+rea_i)}{2}+rea_j-1\}} PM_{ij} = 55 < \infty$,
rv=55 and save *opt_rea*=[1,2,3,5,4];
UN={4,5};
UN={3,4,5};
E={3,**5**};
EN={3,4}, *UN*={3,4} and rea_5=3;
EN={3,4};
RECURSIVE_ENUMERATION(4, {3,4});
E={**3**};
EN={4}, *UN*={4} and rea_3=4;
EN={4};
RECURSIVE_ENUMERATION(5, {4});
rea_4=5;
Since p=5 and $\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} 2^{\{n(rea_i-1)-\frac{rea_i(1+rea_i)}{2}+rea_j-1\}} PM_{ij} = 55 = rv$, continue;
UN={3,4};
UN={3,4,5};
UN={2,3,4,5};
UN={1,2,3,4,5};
Return;

Rearrange the precedence matrix *PM*. Since *opt_rea*=[1,2,3,5,4] we have to switch row 4 and row 5 as well as column 4 and column 5 in order to obtain the standardized precedence matrix *SPM* as shown in Fig. 3.

	2	3	4	5
1	1	1	-	1
2		1	-	1
3			-	-
4				-

(a)

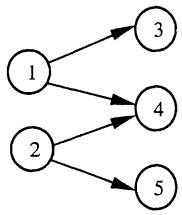
→ unique representation(*AN*)→

	2	3	4	5
1	1	1	1	-
2		1	1	-
3			-	-
4				-

(b)

Fig. 3. (a) Original precedence matrix *PM* and **(b)** its standardized version *SPM* after applying the unique representation algorithm

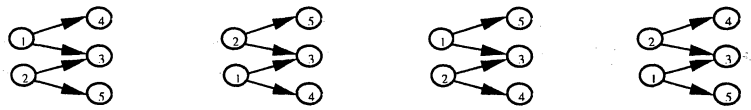
In the example above, two possible rank orders, as given by their recursively enumerated arrays *rea*, have been found. Notice that the representation value for both arrays is equal to 55. However, this is not always the case, as we will show by means of the network given in Fig. 4. The network consists of 5 nodes and two dummy nodes.



	2	3	4	5
1	-	1	1	-
2		-	1	1
3			-	-
4				-

Fig. 4. An example network with its corresponding precedence matrix *PM*

During the recursive enumeration procedure, a number of representations will be found resulting in a value for its corresponding array *rea*. As shown in Fig. 5, the matrices (a) *rea*=[1,2,4,3,5] and (b) *rea*=[2,1,5,3,4] have a representation value *rv*=86 while the representation value of the matrices (c) *rea*=[1,2,5,3,4] and (d) *rea*=[2,1,4,3,5] equals *rv*=58. Notice that these triangular matrices correspond to a network with the topological structure shown in Fig. 4. The corresponding *PM*, however, will never be enumerated as in the recursion, node 4 will always be considered before nodes 3 and 5. The enumeration procedure will store the representation with *rv*=58.



	2	3	4	5
1	-	1	1	-
2		1	-	1
3			-	-
4				-

(a) *rv* = 86

	2	3	4	5
1	-	1	1	-
2		1	-	1
3			-	-
4				-

(b) *rv* = 86

	2	3	4	5
1	-	1	-	1
2		1	1	-
3			-	-
4				-

(c) *rv* = 58

	2	3	4	5
1	-	1	-	1
2		1	1	-
3			-	-
4				-

(d) *rv* = 58

Fig. 5. Four networks with the same topological structure with their corresponding precedence matrices *PM*

3.2 A dominance rule for the recursive algorithm

In order to improve the efficiency of the enumeration algorithm, we use a modified definition of the auxiliary variable E used by the recursive enumeration method. Instead of simply including the nodes $j \in EN$ with the highest weight w_j into the set E, we expand the condition as follows:

$$E=\{i|w_i=\max(w_j|j\in EN),\ \neg\exists j\in E\backslash\{i\}|\ \overline{P}_i=\overline{P}_j \text{ and } \overline{S}_i=\overline{S}_j\};$$

The first condition, $w_i=\max(w_j|j\in EN)$, searches for the nodes with the maximum weight w_j as described above. The second condition, $\neg\exists j\in E\backslash\{i\}|\overline{P}_i=\overline{P}_j$ and $\overline{S}_i=\overline{S}_j$, assures that no node with the same successors and predecessors as an already included node will be selected to enter the set E.

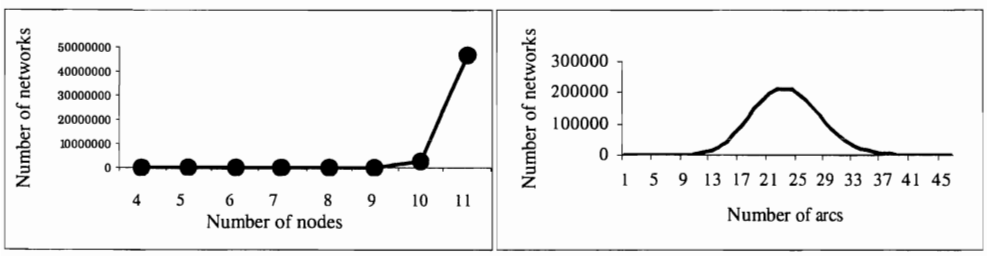
Consider again the example of Fig. 1 as described above. The new definition of the auxiliary variable E allows for a considerable reduction of the search effort. At level three of the enumeration (denoted by *RECURSIVE_ENUMERATION*(3, {3,4,5})) we created the set $E=\{3,5\}$ since both node 3 and node 5 have a maximum weight of 513 among the nodes of *EN*. With the new definition things are different: both node 3 and node 5 are still eligible to enter the set E. But once a node has been selected (e.g. node 3), the other node (node 5) does no longer satisfy the second condition, since it has the same set of successors and predecessors as an already selected node (node 3) of the set E. This results in the fact that one activity (3 or 5) will be randomly chosen and that we will find only one *rv* with the same value as found earlier.

In the next section we explain the logic of the generation method used in *RanGen*. During the generation of the networks, the recursive enumeration method of Section 3.1 will be used in order to prevent the generation of two networks with the same topological structure.

4 Generation method

4.1 The exhaustive generation of strongly random activity networks

It might be tempting to generate strongly random networks (Demeulemeester et al., 1993) by enumerating all possible network structures which satisfy preset values of the complexity parameters. The results of such an enumeration effort are shown in Fig. 6. Unfortunately, as shown in the figure, both CPU-time and memory requirements render such a method inapplicable for networks with more than 10 activities.



(a) (b)

Fig. 6. The impact of the number of activities **(a)** and the number of precedence relations **(b)** on the number of possible network structures

Fig. 6 (a) displays the number of different AoN networks for a number of nodes ranging from 4 to 11. Fig 6 (b) shows the number of different 10-activity networks as a function of the number of precedence relations. While there are somewhat less than 50,000,000 different networks containing 11 nodes, memory restrictions prohibited us from finding the exact number of networks with 12 or more nodes. Remark also that there are many more networks with an *OS*=0.5 (i.e. either 22 or 23 arcs) than e.g. an *OS*=0.25. Since we need one bit for the representation of a precedence constraint and the maximum number of precedence relations is $n(n\text{-}1)/2$, we need 6 bytes of memory to store a network of 10 activities, 24 bytes to store a 20-activity network and 55 bytes to store a 30-activity network. Suppose we have 64 Mb RAM available and we want to enumerate all possible networks containing 30 activities, then we will run out of memory after 1,220,161 networks, which is only a very small fraction of the full space of possible networks. Due to these reasons, we were obliged to search for another generation method, which will be discussed in the next section.

4.2 The *RanGen* procedure

RanGen imposes both a CPU time limit and a limit on the number of networks which may be generated. As many networks as possible are generated within these limits and a number of networks satisfying the preset parameter values are then randomly generated from the obtained set. Using *GN* to denote the set of already generated networks and *AN* to denote a generated activity network in AoN format, the overall procedure can be written in pseudocode as follows:

```
procedure generate(OS);
GN=∅;
Repeat
    AN=remove_arcs(OS);
    Unique_representation(AN);
    If (AN∉GN) then
        Save the network: GN=GN∪{AN};
        Transform AN into an AoA format;
        Compute CI;
Until bound or time limit;
Select a number of networks with prespecified OS and CI;
Return;
```

4.2.1 The procedure remove_arcs(*OS*): generating a network with prespecified order strength

The generator starts for each generation with a completely connected network with *OS*=1 and removes non-transitive arcs until it obtains a network with specified order strength. The set of non-transitive arcs is updated each time an arc is removed. Fig. 7 shows the different steps needed to generate the network of Fig. 1.

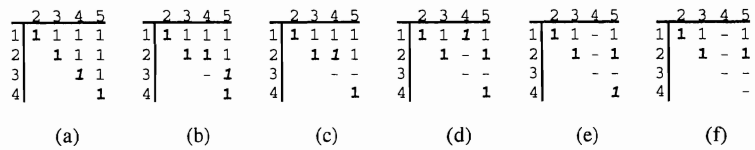


Fig. 7. remove_arcs(0.5): generating the network of Fig. 1

Fig 7 (a) displays the completely connected network for which *OS*=1. The four (in general (*n*-1)) non-transitive direct arcs which are *eligible* for selection are shown in bold. We randomly select arc (3,4) and remove it from the precedence matrix *PM*. In updating the set of direct arcs, both arc (2,4) and arc (3,5) are made eligible for selection (the corresponding 1's are shown in bold in Fig. 7 (b)). Again, we randomly choose an eligible arc (3,5) and update the set of eligible arcs as given in Fig. 7 (c). Since the maximal number of arcs equals 10 and we search for a network with an *OS*=0.5, we repeat this random selection method five times until we obtain the network in Fig. 7 (f) with the given *OS*.

Observe that the generation of a network with prespecified *OS*-value boils down to the deletion of arcs. This simple logic yields *RanGen* a competitive advantage over the existing generators *ProGen* and *ProGen/Max*, which do not allow the generation of networks with small *OS*-value.

4.2.2 Checking for uniqueness: procedure Unique_representation(*AN*)

Upon the generation of a network with preset *OS*-value, it must be checked for uniqueness by the recursive enumeration procedure described in section 3. If the network is new, it is added to the set of generated networks *GN* (initially *GN*=∅).

4.2.3 Compute the complexity index *CI*

Each time a new activity network *AN* has been generated, its *CI*-value must be calculated using the algorithm developed by Bein et al. (1992). Since this algorithm works on networks in AoA format, we first have to transform the generated AoN network into an equivalent AoA network with minimal complexity index using the algorithm of Kamburowski et al. (1992). Notice that the probability of finding extreme values for the complexity index *CI* depends on the number of iterations in the *RanGen* procedure (either a CPU time limit or a limit on the number of networks). An indication of these extreme values is shown in Table 4 of section 5.

4.2.4 The random selection of networks

Upon completion, the program yields a set of networks which satisfy preset *OS*- and *CI*-values from which a desired number of networks may be randomly selected.

4.2.5 Truncating procedure Unique_representation(*AN*)

For the generation of networks with a small order strength *OS* (i.e. for $OS \leq 0.2$, as obtained from our experiments), the recursive enumeration procedure Unique_representation(*AN*) needs a large amount of time to compute the smallest representation value *rv*. Therefore, we provide the option to truncate the search whenever a certain number of matrices *rea* are found. Although there still is a good chance that the optimal rank order, denoted by its array *opt_rea*, will be found after a limited number of searching steps, this method does no longer assure to find a unique representation of each network.

In the following section we elaborate on the relation between the order strength *OS* and the complexity index *CI*.

5 Complexity index *CI* and order strength *OS*: an overview

The network generator *RanGen* is developed for generating different networks which allows the user to perform several validation experiments and to determine the effectiveness and efficiency of different project scheduling procedures in relation to several complexity measures. Since we claim to generate networks with prespecified values for both *OS* and *CI*, realistic input values for both parameters must be entered in order to allow the generator to obtain sufficient problem instances during generation. These values must be selected with sufficient care, as shown in the following example. Suppose a user wants to validate an algorithm by means of a full factorial experiment on a randomly generated dataset. Assume *RanGen* is provided with two settings for the number of activities, three settings for the order strength *OS* and four settings for the complexity index *CI* as given in Table 2.

Table 2. Parameters settings for the example dataset

Number of activities	10 or 20
Order strength *OS*	0.25, 0.50 or 0.75
Complexity index *CI*	3, 6, 9 or 12

Although it is perfectly possible to generate networks with 10 activities, $OS=0.25$ and $CI=3$, *RanGen* will not be able to generate networks of 10 activities with e.g. $OS=0.25$ and $CI=12$. Apparently, the range for the complexity index was chosen too large to allow for the generation of a dataset with the preset parameter values.

In order to provide the user of *RanGen* with a guideline for presetting the input parameters in a full factorial experiment, we have set up the following experiment which provides additional insight in the relation between the *OS* and the *CI*. We have generated a large number of networks with different input parameters for both the number of activities and the order strength as given in Table 3. For each network, we have calculated the complexity index *CI*. The input parameters are set up as follows: the number of activities is varied from 5 to 120 in steps of 5, resulting in 24 settings while the *OS* has 20 settings, varying between 0.05 and 1.00 in steps of 0.05. We have generated networks with a ½ hour time limit for each class (on a Dell personal

computer (Pentium III 800 MHz processor)). Consequently, the experiment took 10 days of CPU-time.

Table 3. Parameters settings for the experiment

Number of activities	5, 10, 15, 20, ... or 120
Order strength *OS*	0.05, 0.10, 0.15, ... or 1.00

Table 4 summarizes the results found from the experiment. This table displays the *CI*-values that were found for each setting of the order strength *OS* and the number of activities. We only list the *CI* for which 10 or more instances were found.

Table 4. Complexity index *CI* and order strength *OS* : An overview

	*OS**																			
	0,05	**0,10**	**0,15**	**0,20**	**0,25**	**0,30**	**0,35**	**0,40**	**0,45**	**0,50**	**0,55**	**0,60**	**0,65**	**0,70**	**0,75**	**0,80**	**0,85**	**0,90**	**0,95**	**1,00**
10		[0,0]	[0,2]	[0,3]	[0,4]	[0,5]	[0,5]	[0,5]	[0,5]	[0,6]	[0,6]	[0,6]	[0,6]	[0,6]	[0,6]	[0,5]	[0,4]	[0,2]	[0,0]	
15	[0,1]	[0,4]	[0,6]	[0,7]	[1,7]	[1,8]	[1,8]	[2,9]	[1,9]	[2,9]	[1,10]	[1,10]	[0,10]	[0,10]	[0,10]	[0,11]	[0,11]	[0,8]	[0,3]	
20	[0,3]	[1,7]	[2,8]	[3,9]	[3,10]	[4,11]	[4,11]	[4,12]	[5,12]	[5,13]	[4,14]	[4,14]	[3,14]	[2,15]	[2,15]	[1,15]	[0,14]	[0,13]	[0,7]	
25	[0,6]	[2,9]	[3,11]	[5,12]	[6,13]	[7,14]	[7,15]	[7,15]	[8,16]	[8,17]	[8,17]	[7,18]	[7,18]	[6,19]	[5,19]	[4,19]	[2,19]	[0,17]	[0,10]	
30	[1,7]	[4,11]	[5,14]	[7,15]	[8,16]	[9,17]	[10,18]	[11,19]	[11,20]	[11,21]	[11,21]	[11,22]	[11,22]	[10,23]	[9,24]	[8,24]	[6,23]	[2,21]	[0,13]	
35	[2,9]	[5,14]	[8,16]	[9,18]	[11,19]	[12,20]	[13,21]	[14,23]	[15,24]	[15,24]	[15,25]	[15,26]	[15,27]	[14,27]	[13,28]	[11,28]	[9,28]	[5,25]	[0,17]	
40	[3,12]	[7,16]	[10,19]	[11,21]	[13,22]	[15,24]	[16,25]	[17,26]	[18,27]	[19,28]	[19,29]	[19,30]	[19,31]	[19,32]	[17,32]	[16,33]	[13,32]	[8,31]	[2,22]	
45	[5,13]	[9,19]	[12,21]	[14,23]	[16,26]	[18,27]	[19,28]	[21,30]	[21,31]	[22,32]	[23,33]	[23,34]	[24,35]	[23,36]	[22,37]	[21,37]	[18,37]	[12,36]	[4,27]	
50	[6,15]	[11,21]	[14,24]	[17,26]	[19,29]	[20,30]	[22,32]	[24,33]	[25,35]	[26,36]	[27,37]	[27,38]	[28,39]	[28,40]	[27,41]	[25,42]	[22,42]	[17,40]	[7,31]	
55	[7,18]	[13,23]	[17,27]	[19,29]	[22,32]	[24,34]	[25,35]	[27,37]	[28,38]	[30,40]	[31,41]	[32,42]	[33,44]	[33,45]	[32,46]	[30,47]	[27,47]	[22,45]	[10,36	
60	[9,20]	[15,26]	[19,30]	[22,32]	[24,35]	[27,37]	[28,39]	[30,41]	[32,42]	[34,44]	[35,45]	[37,46]	[37,48]	[37,49]	[37,50]	[35,51]	[33,52]	[26,50]	[13,41]	
act 65**	[11,22]	[17,28]	[22,32]	[25,35]	[28,38]	[30,40]	[32,42]	[34,44]	[36,46]	[38,47]	[39,49]	[40,51]	[42,52]	[42,53]	[42,55]	[40,56]	[36,56]	[32,55]	[17,45]	
70	[12,24]	[20,31]	[24,35]	[28,38]	[31,41]	[33,43]	[35,45]	[37,47]	[39,49]	[41,51]	[43,53]	[45,55]	[46,56]	[47,58]	[47,59]	[45,60]	[43,61]	[35,60]	[21,50]	
75	[14,26]	[22,33]	[27,37]	[31,41]	[34,44]	[36,47]	[39,49]	[41,51]	[43,53]	[45,55]	[47,57]	[49,58]	[51,60]	[51,62]	[52,63]	[51,65]	[48,66]	[41,65]	[25,55]	
80	[16,28]	[24,35]	[29,40]	[34,44]	[36,47]	[40,50]	[42,52]	[44,55]	[47,57]	[49,59]	[51,61]	[53,62]	[55,64]	[56,66]	[56,68]	[56,69]	[53,70]	[46,70]	[29,60]	
85	[18,30]	[26,38]	[32,43]	[36,47]	[40,50]	[42,53]	[45,56]	[48,58]	[50,60]	[53,62]	[55,64]	[57,66]	[59,68]	[61,70]	[62,72]	[61,74]	[59,75]	[52,74]	[34,65]	
90	[19,32]	[29,40]	[35,45]	[39,49]	[43,53]	[46,56]	[48,59]	[51,62]	[54,64]	[57,66]	[59,68]	[61,70]	[63,72]	[65,74]	[66,76]	[66,78]	[64,80]	[57,79]	[38,70]	
95	[22,34]	[31,43]	[37,48]	[42,52]	[46,56]	[49,59]	[52,63]	[55,65]	[58,68]	[60,70]	[63,72]	[65,74]	[68,77]	[70,78]	[71,81]	[71,83]	[69,84]	[63,84]	[43,74]	
100	[24,36]	[33,45]	[40,51]	[45,55]	[49,60]	[53,63]	[56,66]	[58,69]	[61,71]	[64,74]	[67,76]	[69,78]	[72,81]	[74,83]	[75,85]	[76,87]	[74,89]	[67,89]	[47,80]	
105	[26,39]	[36,48]	[43,54]	[48,58]	[52,62]	[56,66]	[60,69]	[62,72]	[65,75]	[69,77]	[71,80]	[74,82]	[76,85]	[78,87]	[80,89]	[81,92]	[80,93]	[73,94]	[54,84]	
110	[27,41]	[39,50]	[46,57]	[51,61]	[56,65]	[60,69]	[62,73]	[66,76]	[69,79]	[72,82]	[75,84]	[78,86]	[80,89]	[83,91]	[85,94]	[86,96]	[84,98]	[79,99]	[59,89]	
115	[29,43]	[41,53]	[48,59]	[54,64]	[58,68]	[63,72]	[66,76]	[69,79]	[73,82]	[76,85]	[79,88]	[82,90]	[85,93]	[88,96]	[90,98]	[90,100]	[90,103]	[85,103]	[63,94]	
120	[33,44]	[44,55]	[52,62]	[57,67]	[62,71]	[68,75]	[71,79]	[74,82]	[78,85]	[81,88]	[84,91]	[86,95]	[89,97]	[92,100]	[94,102]	[96,105]	[95,107]	[90,108]	[69,99]	

* OS : Order Strength

** act: Number of activities

In the next section we review a number of resource measures from the literature and their implementation in *RanGen*.

6 Resource measures

Several resource measures have been introduced in the literature to describe the relation between the presence of different resources and their impact on the hardness of a project scheduling instance. Patterson (1976) has listed a large number of resource utilization parameters reflecting the tightness of certain resource types as well as different constrainedness parameters. Other famous attempts in describing resource parameters have been made by Pascoe (1966), Cooper (1976), Alvarez-Valdes and Tamarit (1989) and Kolisch et al. (1995).

In generating the resource measures for a project several decisions have to be made. First, we determine the density of the resource demand matrix in order to specify whether an activity uses a particular resource or not. This is done by computing the resource factor *RF* and the resource use *RU*. Secondly, the resource demand and resource availability for each activity are generated. Therefore, we either determine the resource strength *RS* or the resource constrainedness *RC*. In the following we make a distinction between the single-mode and the multi-mode case.

6.1 Single-mode case

ProGen (Kolisch et al., 1995) and *ProGen/Max* (Schwindt, 1995) use the **resource factor** *RF*. This parameter, introduced by Pascoe (1966) and utilized in studies by Cooper (1976) and Alvarez-Valdes and Tamarit (1989), can be calculated as follows:

$$RF = \frac{1}{nK} \sum_{i=1}^{n} \sum_{k=1}^{K} \begin{cases} 1, \text{if } r_{ik} > 0 \\ 0, \text{otherwise} \end{cases} \qquad [1]$$

where n denotes the number of activities (excluding dummy activities), K denotes the number of resource types and r_{ik} denotes the amount of resource type k required by activity i. The resource factor *RF* reflects the average portion of resource types requested per activity and consequently measures the density of the matrix r_{ik}. According to Kolisch et al. (1995), there is a positive relation between the required CPU-time to solve the well-known resource-constrained project scheduling problem and the *RF* while Alvarez-Valdes and Tamarit (1989) have observed that problems with RF=1.0 were easier to solve than problems with RF=0.5.

However, a few remarks should be made on the use of *RF* as a measure of the density of the matrix r_{ik}. When implementing the *RF* as defined in [1], it is possible that no resource requirement will be generated for some activities. This is always the case when $RF < 1/$(number of resources), but it can also happen in other cases (e.g. RF=0.5 and half of the number of activities use all resource types while the other half do not require any resources).

ProGen/Max (Schwindt, 1995) uses a lower bound equal to 1/(number of resources) for *RF* and assures that all activities use at least one resource type. We instead, keep

the original definition of *RF* as given in [1] and implemented a new measure of resource density, the **resource use** *RU*. The resource use *RU* varies between zero and the number of resource types available and measures for each activity the number of resource types used in the following way:

$$RU_i = \sum_{k=1}^{K} \begin{cases} 1, \text{if } r_{ik} > 0 \\ 0, \text{otherwise} \end{cases} \qquad i = 1,...,n \qquad [2]$$

In *RanGen*, RU_i=*RU*, where *RU* is a positive constant, to assure that each activity uses exactly *RU* resource types. Moreover, the impact of the number of resources on problem hardness can also be studied by varying the number of resource types *K* for the set of networks with *RF*=1 (or *RU* equal to the number of resource types).

RanGen relies on two resource measures for generating the resource availability: the resource strength *RS* and the resource-constrainedness *RC*. The **resource strength** *RS* was first introduced by Cooper (1976) and later used by Alvarez-Valdes and Tamarit (1989). We use the new definition introduced by Kolisch et al. (1995):

$$RS_k = \frac{a_k - r_k^{\min}}{r_k^{\max} - r_k^{\min}} \qquad [3]$$

where a_k denotes the total availability of renewable resource type k, $r_k^{\min}$ equals $\max_{i=1,...,n} r_{ik}$ and $r_k^{\max}$ denotes the peak demand of resource type k in the precedence preserving earliest start schedule. Elmaghraby and Herroelen (1980) were the first to conjecture that the relationship between the complexity of a resource-constrained project scheduling problem and the resource availability varies according to a bell-shaped curve. De Reyck and Herroelen (1996) and Herroelen and De Reyck (1999) confirmed this conjecture and rejected the negative correlation between problem difficulty and the *RS* found by Kolisch et al. (1995).

The **resource-constrainedness** *RC*, has been introduced by Patterson (1976):

$$RC_k = \frac{\bar{r}_k}{a_k} \qquad [4]$$

where a_k is defined as above and $\bar{r}_k$ denotes the average quantity of resource type k demanded when required by an activity, i.e. $\bar{r}_k = \sum_{i=1}^{n} r_{ik} \Big/ \sum_{i=1}^{n} \{1, \text{if } r_{ik} > 0; 0 \text{ otherwise}\}$.

The arguments for using either the resource strength *RS* or the resource-constrainedness *RC* are often confounding. Kolisch et al. (1995) argue that the major advantage of the *RS* lies in the incorporated information about the precedence structure of the network, while De Reyck and Herroelen (1996) find this a drawback since then it cannot be considered as a pure measure anymore. In addition, these authors have shown occasions where the *RS* can no longer distinguish between easy

and hard problem instances while *RC* continues to do so. Furthermore, Herroelen and De Reyck (1999) restrict the use of these parameters to the case where there is only one resource type or when *RS* and *RC* are constant over all resources.

In summary, *RanGen* needs a number of inputs for the resource measures. First, we need to specify the number of resource types K and a value of either the *RF* or the *RU*. Alternatively, we can choose to vary the number of resources in an interval while, consequently, the *RF* is set automatically to one (of *RU* to the number of resource types). Secondly, a value for either the *RS* or the *RC* is needed in order to generate the resource availability.

6.2 Multi-mode case

In section 6.1 we assumed a project has only one way of performing each activity. In what follows, we broaden this scope to the introduction of several execution modes for each activity. In this multi-mode project scheduling problem, the activities possess different execution modes reflecting different ways of performing the activities. Each mode is a tuple denoting an activity duration with corresponding resource demand. The interrelation $r_{ikm}=g_{ikm}(d_{im})$ between the durations of the modes and the resource demand is reflected by two types of functions g_{ikm} (Kolisch et al., 1995). r_{ikm} denotes the resource demand of mode m of activity i for resource type k while d_{im} denotes the duration of mode m of activity i. When the resource demand is a decreasing function of the durations, either a time/cost trade-off or a time/resource trade-off is involved. In the former case, the resource is of the nonrenewable type while in the latter case it is a renewable resource. In resource/resource trade-off problems, the resource demand does not depend on the activity duration. We discuss these two interrelations in the following subsections.

6.2.1 Time/cost and time/resource trade-off functions

If the resource demand is decreasing with the duration of the modes, the function g_{ikm} can be either linear, convex, concave or randomly chosen. In order to capture these four cases, we need as an input an interval $SI=[a,b]$ for the slope connecting two adjacent modes. Starting with a randomly chosen resource demand for the mode with the highest duration (normal duration), we randomly choose a value for the slope $s\in[a,b]$. In doing so, we are able to generate the resource demand for the next mode and we update the interval SI' as follows:

- random: $SI'=[a,b]$
- linear: $SI'=[s,s]$
- convex: $SI'=[s,b+s-a]$
- concave: $SI'=[a-b+s,s]$

We repeat this stepwise generation method until the crash duration is reached, which corresponds to a maximum allocation of resources.

Suppose we generate a convex time/resource trade-off function for an activity i with four modes with corresponding durations $d_{i1}=10$ (normal duration), $d_{i2}=9$, $d_{i3}=7$ and

d_{i4}=5 (crash duration). The slope interval SI=[1,3]. The randomly chosen resource demand for the normal duration equals 3, i.e. mode$_1$=(10,3). We select randomly a slope s=2$\in$[1,3], update the interval SI'=[2,4] and determine the next mode as follows: mode$_2$=(9,5). Again, we select randomly slope s equals 2$\in$[2,4]. The updated SI'=[2,4] and the new mode$_3$=(7,9). The last randomly chosen slope s equals 3$\in$[2,4] which results in an updated SI'=[3,5] and the crash mode$_4$=(5,15). The four modes for activity i and one renewable resource type k, $\{(d_{im},r_{ikm})\}$={(5,15),(7,9),(9,5),(10,3)}, clearly represent a convex time/resource trade-off function. In the case of a nonrenewable resource k, the four modes represent a time/cost trade-off function.

6.2.2 Resource/resource trade-off functions

If the resource demand is independent of the duration, the resource requirement for each activity is randomly generated as follows. We assign to each resource type k a weight wr_k and to each activity a total resource value $V_i = \sum_{k=1}^{K} wr_k r_{ikm}$. During the generation of the resource demand for each mode m of activity i, the total resource value V_i is held constant. To that purpose, we randomly generate resource demands for K-1 resource types of the first mode and fix the demand of the last resource type such that the total resource value is held constant. During the generation of the other modes, we increase the demand for a randomly chosen resource type and subsequently determine the demands for the other resource types. We repeat this generation method until all the modes of the activities contain resource demands for each resource type, as will be illustrated in the following example.

Suppose we generate a resource/resource trade-off function of an activity i with three renewable resource types and three modes. The total resource value V_i equals 100 and the weights of the resources are wr_1=10, wr_2=3 and wr_3=5. First, we randomly generate two (K-1) numbers for the resource demands of the first mode, r_{i11}=3 and r_{i21}=5 and subsequently, r_{i31}=(100-3*10-5*3)/5=11. We randomly select the resource type 1 and increase the demand for the second mode with a randomly generated number 2. Therefore, the second mode contains following resource demands: r_{i12}=5 and r_{i22}=5 and subsequently, r_{i32}=(100-5*10-5*3)/5=7. In generating the third mode, we now randomly select resource type 2 and increase its demand with 3 units, i.e. r_{i13}=5 and r_{i23}=8 and subsequently, r_{i31}=(100-5*10-8*3)/5=5. The three modes for activity i with three renewable resource types, $\{(r_{i1m},r_{i2m},r_{i3m})\}$ = {(3,5,11),(5,5,7),(5,8,5)}, clearly represent a resource/resource trade-off function.

In order to generate the availabilities a_k for each resource type k, we define the resource strength RS for the multi-mode case as follows:

$$RS_k = \frac{a_k - r_k^{\min}}{r_k^{\max} - r_k^{\min}}$$

where $r_k^{\min}$ equals $\max_{\substack{i=1,\ldots,n \\ m=1,\ldots,M}} r_{ikm}$ and $r_k^{\max}$ denotes the peak demand of resource type k in the precedence preserving earliest start schedule where each activity has a duration

which corresponds to a maximum allocation of resources. Notice that this definition does not correspond to the definition by Kolisch et al. (1995). In their definition, $r_k^{\min}$ equals $\max_{i=1,\ldots,n}\{\min_{m=1,\ldots,M} r_{ikm}\}$. Consequently, in their approach, the feasibility of the problem could not be assured since low values for the *RS* will lead to many infeasible modes, i.e. modes for which the resource demand exceeds the availability a_k.

7 Conclusion

In this paper we discussed the logic of *RanGen*, a new generator of activity-on-the-node (AoN) networks which can be used to represent the underlying project for different classes of project scheduling problems. *RanGen* avoids the shortcomings of existing network generators since it employs the order strength *OS* as well as the complexity index *CI*, which have been shown in previous experiments to serve as reliable predictors of the hardness of various types of project scheduling problems.

We equipped *RanGen* with a number of resource measures taken from the literature. Both single-mode and multi-mode measures have been implemented in order to describe the relation between the presence of different resource types and their impact of problem hardness.

Copies of the computer program are available on request.

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